University of Waterloo
Faculty of Engineering
Department of System Design Engineering

SYDE 556 - Assignment 3
Connecting Neurons

200 University Avenue West
Waterloo, Ontario, Canada
N2L 3G1

Prepared by
Sugandha Sharma
ID: 20401693
userid: s72sharm
4B Electrical Engineering
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1 Decoding from a population

Created a population of 20 LIF neurons representing a 1-dimensional value, and computed a decoderd for them. For parameters, τref=0.002s, τRC=0.02s, the maximum firing rates were chosen randomly from a uniform distribution between 100 and 200Hz, and the x-intercepts are chosen randomly from a uniform distribution between -2 and 2.

a) Tuning Curves

Plot of the tuning curves (firing rate of each neuron for different x values between -2 and 2)

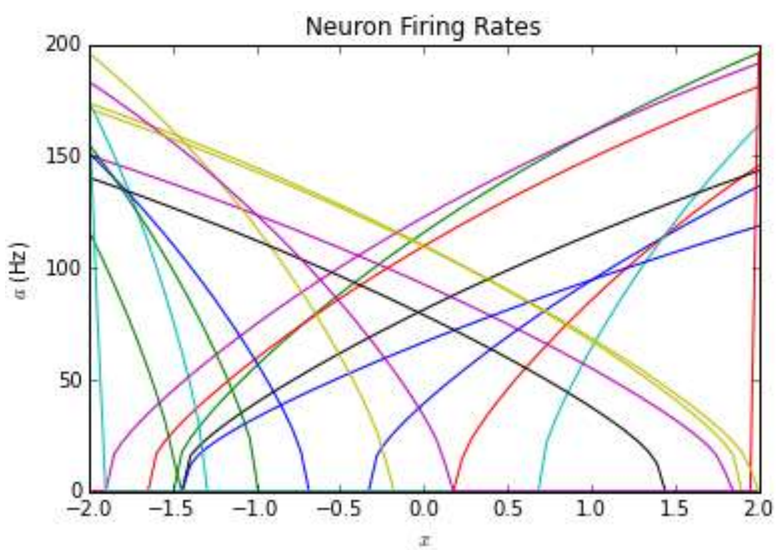

Figure 1. Plot of the tuning curves of 20 LIF neurons

b) Decoders and Plot of x - xhat

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Decoders:
[  1.27662850e-03   1.58774018e-03   1.52336132e-03   2.72047480e-03
  -1.31727968e-03  -2.20702610e-03   1.34301440e-03   5.81915942e-05
  -1.44516505e-03   1.94904299e-03   4.01745452e-04   1.27590367e-03
  -2.64803572e-03  -8.62384047e-04  -6.07390935e-04   1.26390540e-04
   4.31563313e-04  -1.34535833e-04  -1.76196893e-03  -1.40553624e-03]

 RMSE 0.12045362102
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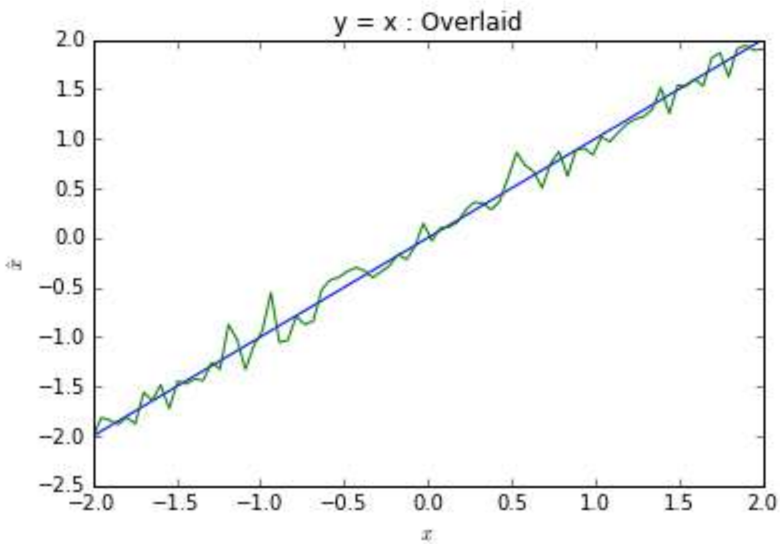

Figure 2. Plot of y = x overlaid

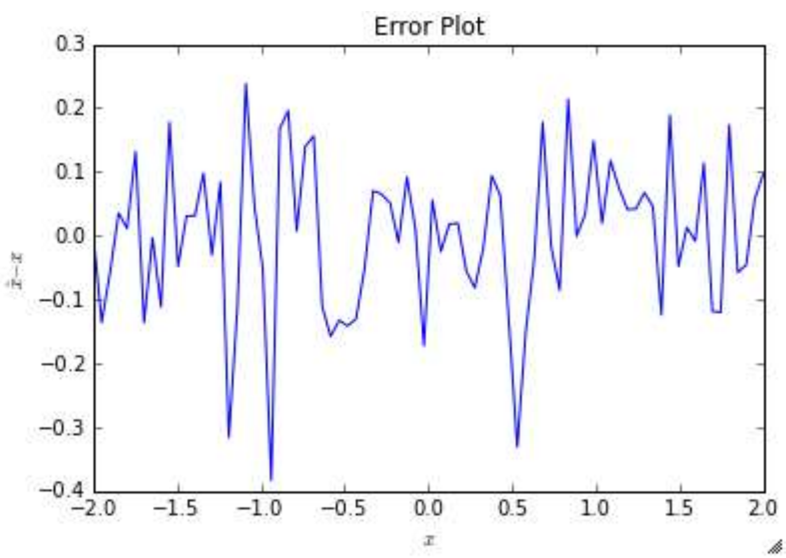

Figure 3. Plot of x - xhat

2 Decoding from two spiking neurons

Chose a neuron from part 1 that has a firing rate of somewhere between 20-50Hz for x=0. Using that neuron's α and Jbias value, constructed two neurons: both with the same α and Jbias, but one with e=+1 and the other with e=-1. Generate a random input x(t) that is 1 second long, with rms=1, dt=0.001, and an upper limit of 5Hz. Fed that signal into the two neurons and generate dspikes. Decoded the spikes back into xhat(t) using a post-synaptic current filter h(t) with a time constant of τ=0.005.

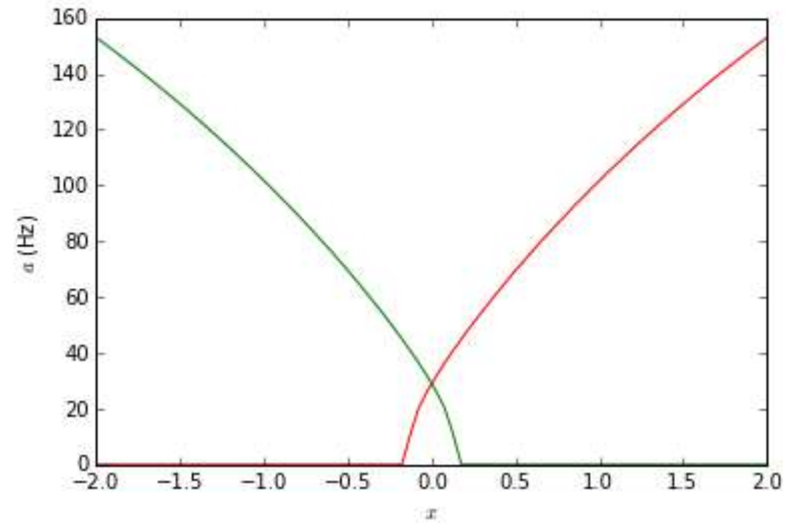

Figure 4. Neurons Chosen with firing rate = 26.350 Hz

a) Post Synaptic Current

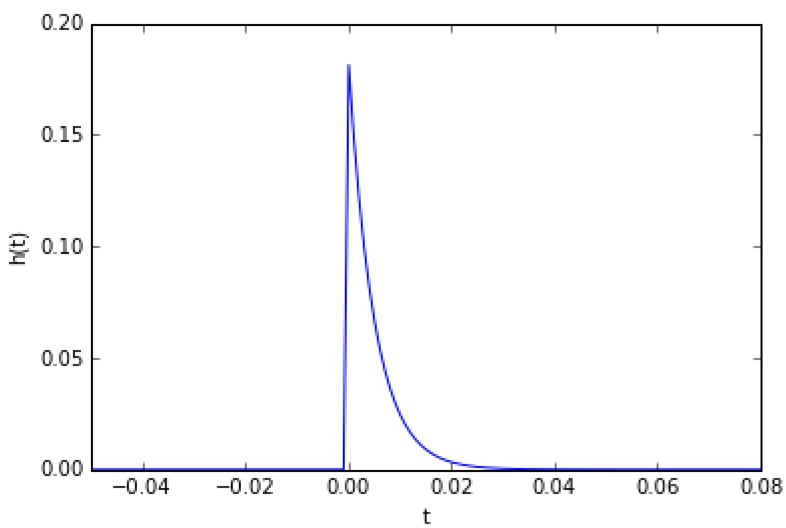

Figure 5. Post Synaptic Current h(t)

b) Reconstruction of the original signal

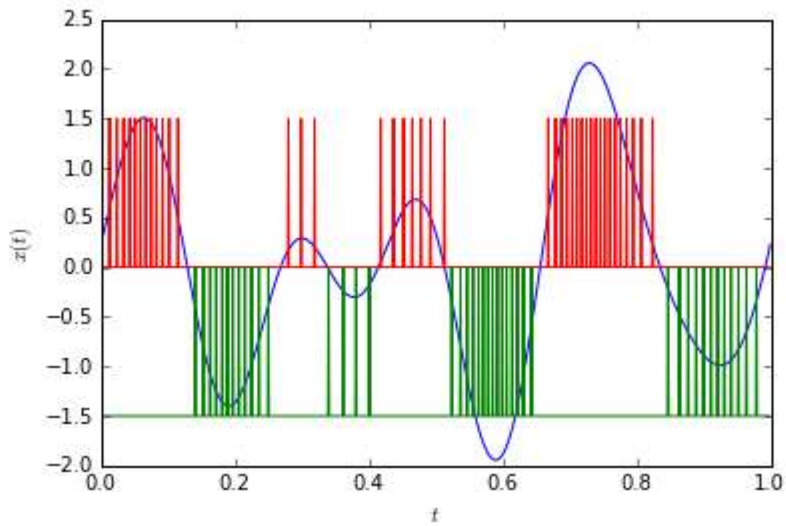

Figure 6. x(t) – blue, spikes e = 1 (red), spikes e = -1 (green)

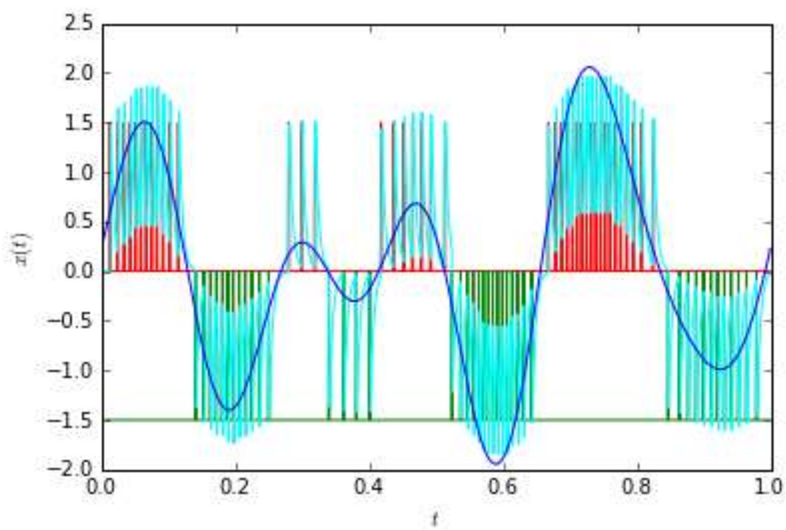

Figure 7. x(t) – blue, xhat(t) – cyan, spikes e = 1 (red), spikes e = -1 (green)

c) Root Mean Square Error of the decoding

RMSE = 0.557216774303

3 Decoding from many neurons

In this part, part 2 was repeated but with more neurons. Instead of picking particular neurons, randomly generated them with x-intercepts uniformly distributed between -2 and 2 and with maximum firing rates between 100 and 200 Hz. Randomly chose encoder values to be either -1 or +1.a)

Tried with 8 neurons, 16 neurons, 32, 64, 128, up to 256 to study the effect on RMSE.

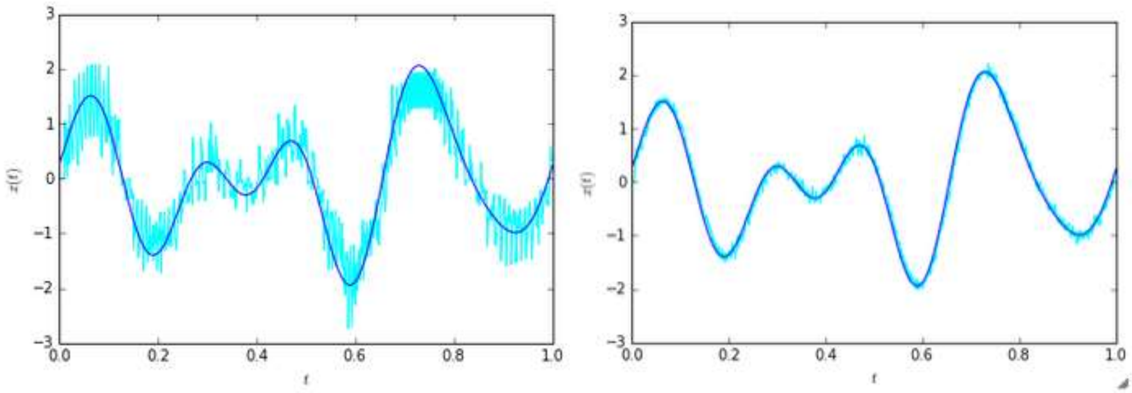

Figure 8. x(t)- blue and xhat(t)- cyan using 8 (left) and 256 (right) number of neurons

a) Plot of RMSE with increasing neurons

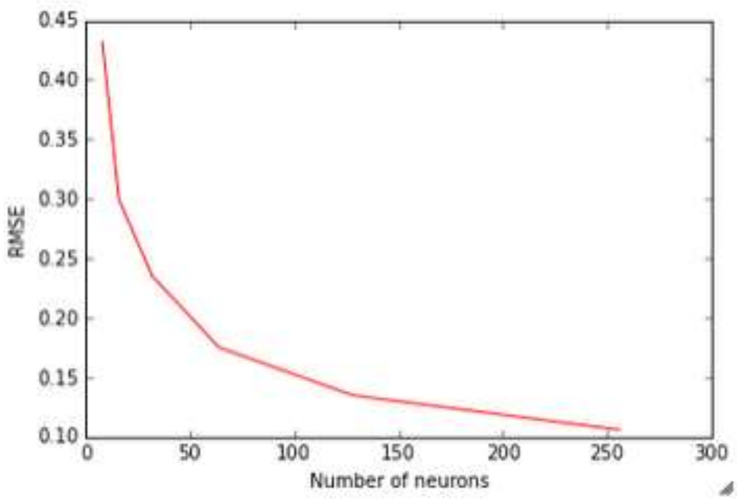

Figure 9. RMSE as the number of neurons increases

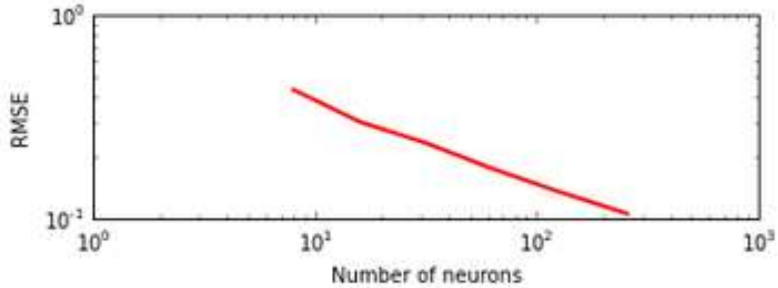

Figure 10. Logplot of RMSE as the number of neurons increases

4 Connecting two groups of neurons

For this question, two groups of neurons were used to compute $y=2x+1$. The first group of neurons represents x and the second group represents y.

a) Behaviour of the system with an input of $x(t)=t-1$

Behaviour of the system with an input of x(t)=t−1 for 1 second (a linear ramp from -1 to 0)

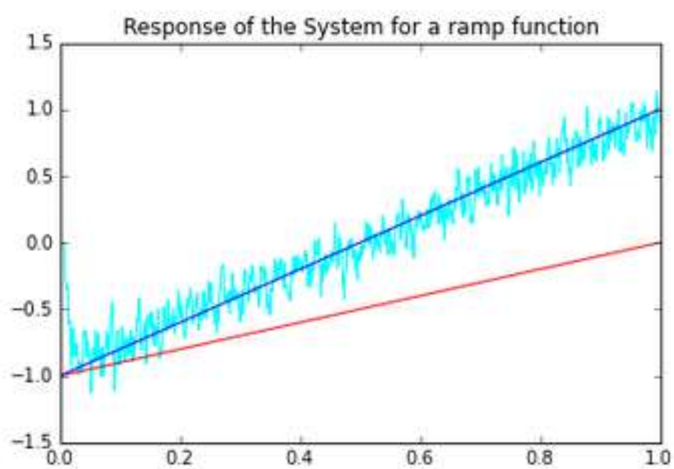

Figure 11. Response of the system to a ramp input [x(t) – red, y(t) – blue, yhat(t) – cyan]

b) Behaviour of the system with a step input

Repeated part (a) with an input that is ten randomly chosen values between -1 and 0, each one held for 0.1 seconds (a randomly varying step input)

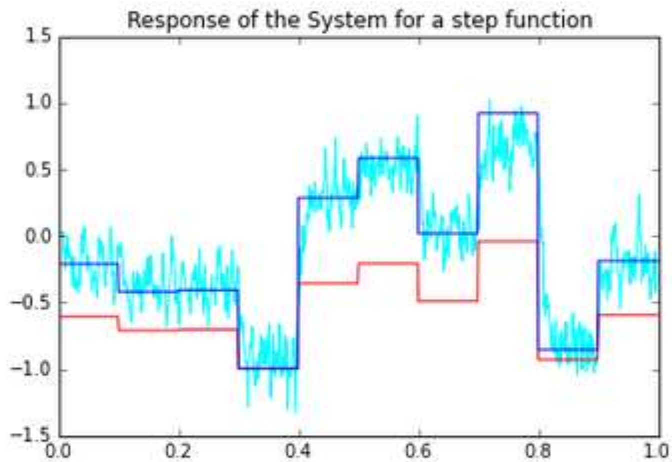

Figure 12. Response of the system to a step input [x(t) – red, y(t) – blue, yhat(t) – cyan]

c) Behaviour of the system with an input $x(t)=0.2sin(6\pi t)$

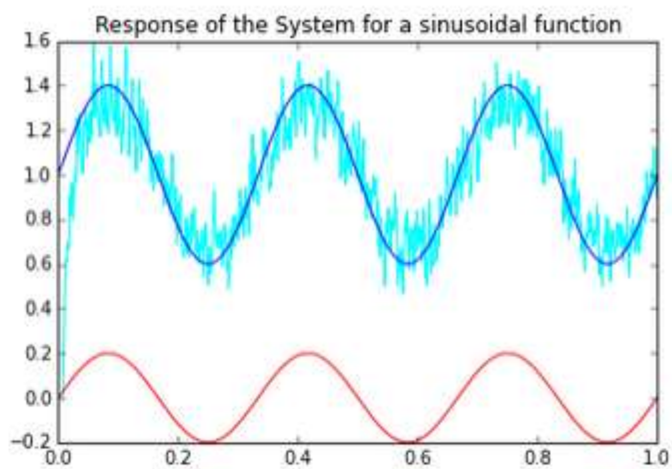

Figure 13. Response of the system to a sinusoid [x(t) – red, y(t) – blue, yhat(t) – cyan]

d) Discussion

From the part a), b) and c), it can be seen that the decoded signal yhat(t) is noisier as compared to the ideal y(t) signal. This is because of the spiky behaviour of the neurons and due to distortion. Also there is a slight time lag between the ideal output y(t) and the decoded output yhat(t) which can be attributed to the propagation delays due to synapses between neurons of the two populations.

On increasing the value of tau from 0.005 to 0.01, the noise is found to decrease as shown in Figure 14. This is because on increasing tau, the filtered signal becomes smoother due to greater averaging of the signal over time. However, this also causes a further time delay in filtering due to which a greater time lag can be seen in Figure 14.

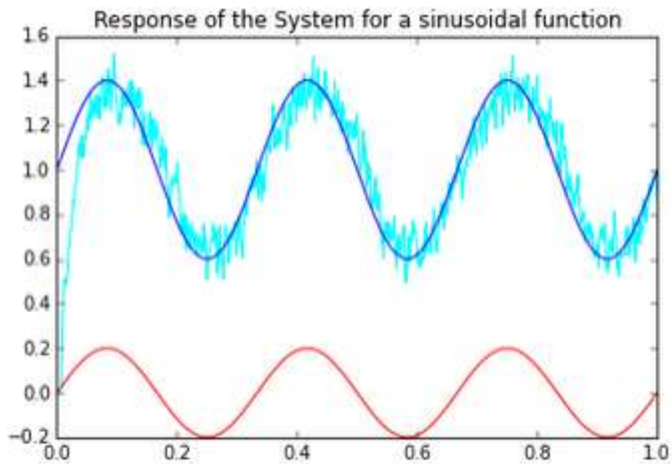

Figure 14. Response on increasing tau to 0.01 [x(t)- red, y(t) -blue, yhat(t)- cyan]

5 Connecting three groups of neurons

For this question, three groups of neurons were used to compute z=2y+0.5x. by taking the decoded outputs from the first two groups of neurons (f(y)=2y and f(x)=0.5x), adding them together, to feed that into the third group of neurons.

a) Response of the system to x(t)=cos(3πt) and y(t)=0.5sin(2πt)

Plot x(t), y(t), the ideal z(t), and the decoded zhat(t) for an input of x(t)=cos(3πt) and y(t)=0.5sin(2πt) (over 1.0 seconds)

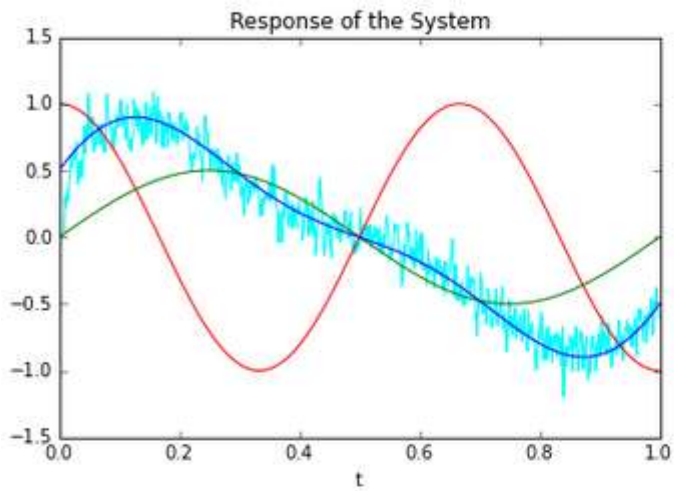

Figure 15. Response of the system [x(t) – red, y(t) – green, z(t) – blue, zhat(t) – cyan]

b) Response of the system to random inputs

Plotted x(t), y(t), the ideal z(t), and the decoded zhat(t) for a random input over 1 second. For x(t) used a random signal with a limit of 8 Hz and rms = 1. For y(t) used a random signal with a limit of 5 Hz and rms = 0.5.

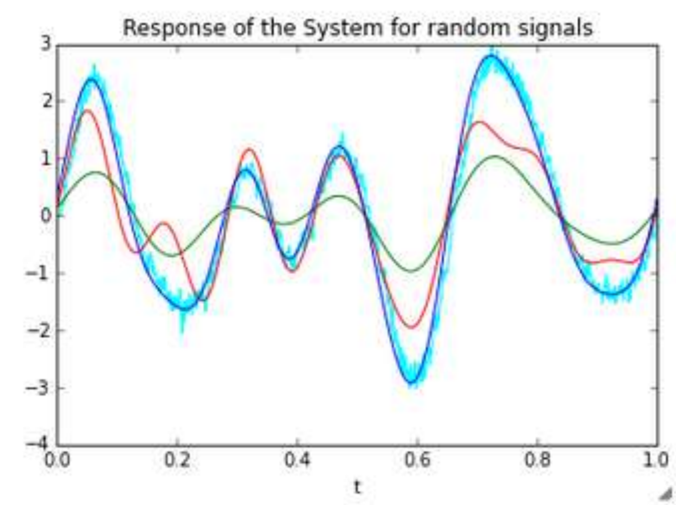

Figure 16. Response of the system [x(t) – red, y(t) – green, z(t) – blue, zhat(t) – cyan]

6 Computing with vectors

Did the same thing as sections 4 and 5, but with 2-dimensional vectors instead of scalars. Everything else was kept the same. For the encoders e, randomly generated them over the unit circle.

The function to compute was $w=x-3y+2z-2q$. This required five groups of neurons: x, y, z, q, and w. Each of them represented a 2-dimensional value. The outputs from x, y, z, and q were all fed into w.

a) Response of the system to constant vector inputs

Plotted the decoded output what(t) and the ideal w for $x = [0.5,1]$, $y = [0.1,0.3]$, $z = [0.2,0.1]$, $q = [0.4,-0.2]$. (Note that these are all constants so they don't change over time, but still plot it for 1.0 seconds).

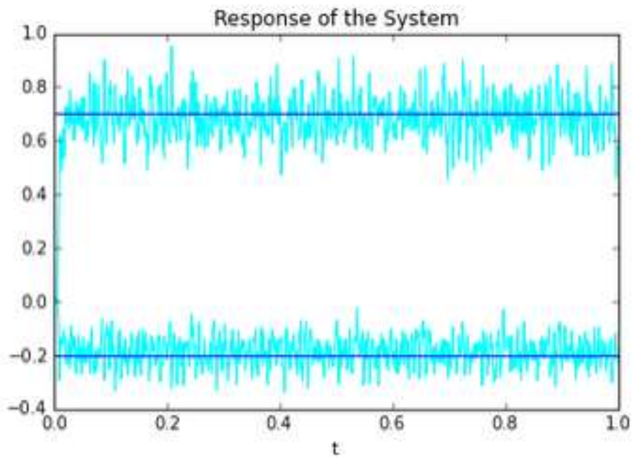

Figure 17. Response of the system [w(t) – blue, what(t) – cyan]

b) Response of the system to varying vector inputs

Produced the same plot as in part a) for $x = [0.5,1]$, $y = [\sin(4\pi t),0.3]$, $z = [0.2,0.1]$, $q = [\sin(4\pi t),-0.2]$.

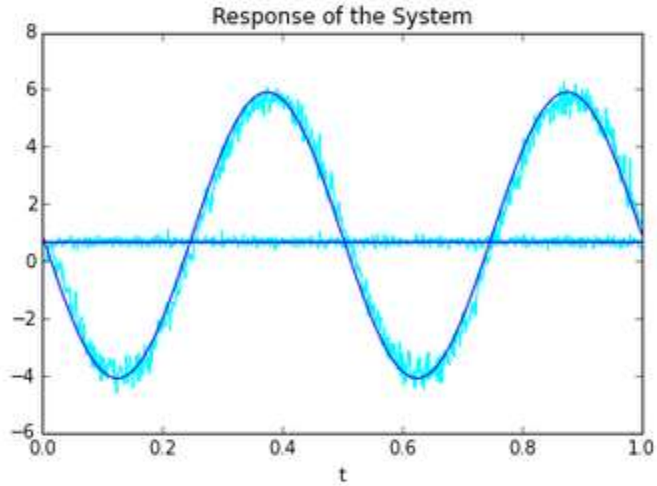

Figure 18. Response of the system [w(t) – blue, what(t) – cyan]

c) Discussion

It can be seen from Figure 17 and Figure 18 that this system does a pretty good job of vector addition/subtraction, however, the decoded output is noisier than the ideal output. There is also a brief transient at the startup (more noticeable in Figure 17) but after that the sum remains quite accurate even under noisy conditions. The noise is because of the spiky behaviour of neurons and due to distortion. Moreover there is a slight time lag between the expected output w(t) and the decoded output what(t) (more noticeable in Figure 18). The cause of this lag is the propagation delays caused by the synapses between the 5 populations.

Each population contains 200 neurons. Increasing the number of neurons should further improve the system's performance.

On increasing the value of tau from 0.005 to 0.01, the noise is found to decrease as shown in Figure 19. This is because on increasing tau, the filtered signal becomes smoother due to greater averaging of the signal over time. However, this also causes a further time delay in filtering due to which a greater time lag can be seen in Figure 19.

Additionally, the spike placement can be improved by generating spikes through interpolation into the time rather than using the difference equation. This should also increase the accuracy of the results obtained.

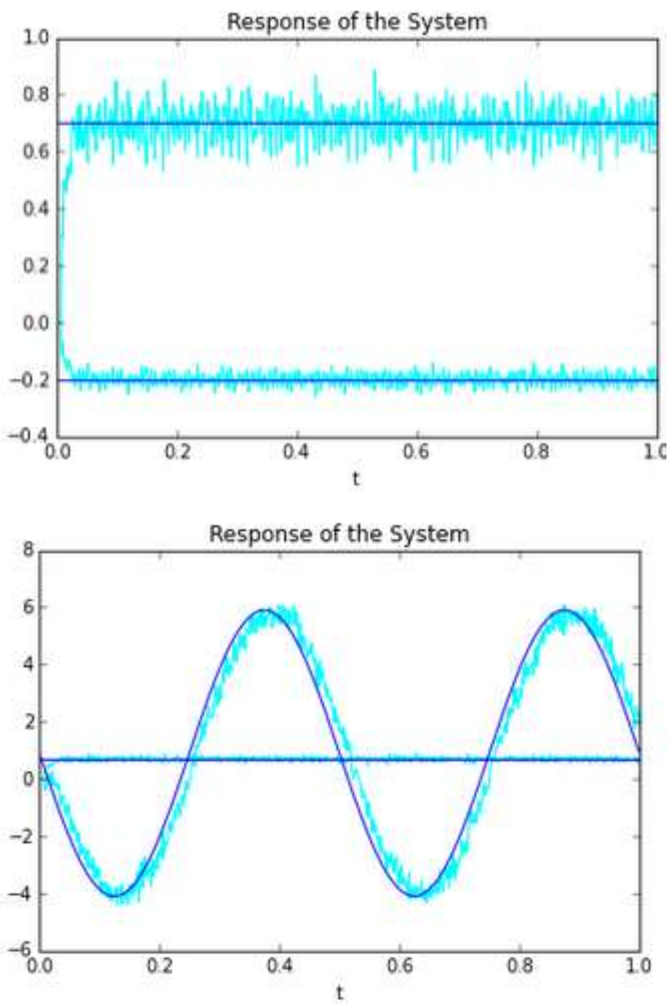

Figure 19. Response of the system with tau = 0.01